Final Report

of

The project entitled

ELASTIC PLASTIC AND CREEP ANALYSIS PROBLEMS IN THE DISC

SANCTIONED FROM

UNIVERSITY GRANTS COMMISSION, NEW DELHI

(MRP-MAJOR-MATH-2013-41603)

(F. No. 43-438/2014(SR) DATED 24.9.2015)

IN

MATHEMATICS

TO

Dr. Satya Bir Singh

Principal Investigator

Department of Mathematics

Punjabi University, Patiala

Dr Pankaj

Co-Investigator

Department of Mathematics

ICFAI University, Baddi

Department of Mathematics

Punjabi University, Patiala-147002

2018

CERTIFICATE

We, Prof. Satya Bir Singh and Dr. Pankaj declare that the work presented in this report is original and carried throughout independently by us during the annual tenure of major research project of U.G.C.; New Delhi.

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Annexure -VIII

Final Report of the work done on the Major Research Project

1. Project report No. : **Final**

2. UGC Reference No. F.: **43-438/2014(SR)**

3. Period of report: **From 01 July 2015 to 30 June 2018**

4. Title of research project: **Elastic Plastic and Creep analysis problem in the Disc**

5. (a) Name of the Principal Investigator : **Dr. Satya Bir Singh (Professor)**

   (b) Deptt. : **Department of Mathematics**

   (c) University/College where work has progressed: **Punjabi University, Patiala**

6. Effective date of starting of the project: **24 September 2015**

7. Grant approved and expenditure incurred during the period of the report:

a. Total amount approved: **Rs. 10, 88, 520/-**

b. Total expenditure: **Rs. 10, 88, 300/-**

c. Report of the work done:

i. Brief objective of the project: **Please refer Appendix 1**

ii. Work done so far and results achieved and publications, if any, resulting from the work **Please refer Appendix IV and VI**

iii. Has the progress been according to original plan of work and towards achieving the objective? If not, state reasons.

Yes, the progress of the project was as per plan.

iv. Please indicate the difficulties, if any, experienced in implementing the project.

The appointment of project fellow was delayed, as the fellow selected did not join. Another project fellow was appointed later on whose date of joining is 28/04/2016

v. If project has not been completed, please indicate the approximate time by which it is likely to be completed. A summary of the work done for the period (Annual basis) may please be sent to the Commission on a separate sheet

Not Applicable as the project has been completed.

vi If the project has been completed, please enclose a summary of the findings of the study. One bound copy of the final report of work done may also be sent to
University Grants Commission.

Please refer Appendix V

vii. Any other information which would help in evaluation of work done on the project. At the completion of the project, the first report should indicate the output, such as

a) **Manpower trained** : The appointed Project fellow has been Registered in Ph.D. and will work on similar topics related to the project
b) **Publication of results:** The various problems which were solved during the project are drafted in form of research papers and published in reputed journals indexed in Thompson Reuters list.

Annexure – IX

PROFORMA FOR SUBMISSION OF INFORMATION AT THE TIME OF SENDING THE FINAL REPORT OF THE WORK DONE ON THE PROJECT

1. Title of the project: **Elastic Plastic and Creep Analysis Problem in the Disc**

2. Name and address of the principal investigator

Dr. Satya Bir Singh, Department of Mathematics, Punjabi University, Patiala, Punjab.

3. Name and address of the Institution: **Punjabi University, Patiala, Punjab, 147002**

4. UGC approval letter no. and date**: F. No. 43-438/2014(SR) DATED 24.9.2015**

5. Date of implementation: **24 September 2015**

6. Tenure of the project: **From 01 July 2015 to 30 June 2018**

7. Total grant allocated: **Rs. 10, 88, 520/-**

8. Total grant received: **Rs. 10, 14, 168/-**

9. Final expenditure: **Rs. 10, 88, 300/-**

10. Title of the project: **Elastic Plastic and Creep Analysis Problem in the Disc**

11. Objectives of the project: **Please refer Appendix 1**

12. Whether objectives were achieved: **Yes**

13. Achievements from the project: **Please refer Appendix IV**

14. Summary of the findings: Please **refer Appendix V**

15. Contribution to the society: The results obtained in this project have a potential impact on the design reliability of machines in heavy as well as light industries.

16. Whether any Ph.D. enrolled/produced out of the project: **The appointed project fellow is registered in Ph.D.**

17. No. of publications out of the project: **Please refer Appendix VI**

Objectives

The objective of this research project to solve elastic, plastic and creep problem analytically as well as computationally. In order to explain the elastic-plastic and creep transition, it is first necessary to recognize the transition state as an asymptotic one. Seth (1962) identified the transition state in which the governing differential equation showed some criticality. The general yield condition given by Seth's (1970) of transition is identified by vanishing of Jacobian of transformation. In this work, it is our main aim to eliminate the need for assuming semi-empirical laws, yield condition, creep-strain laws, jump conditions, etc. Efforts will be made to obtain the constitutive equation corresponding to the transition state and fully plastic state. One of the most interesting results in this work concerns the identification of the transition state. A general treatment of transition in elastic-plastic and creep phenomena will be presented.

Methodology

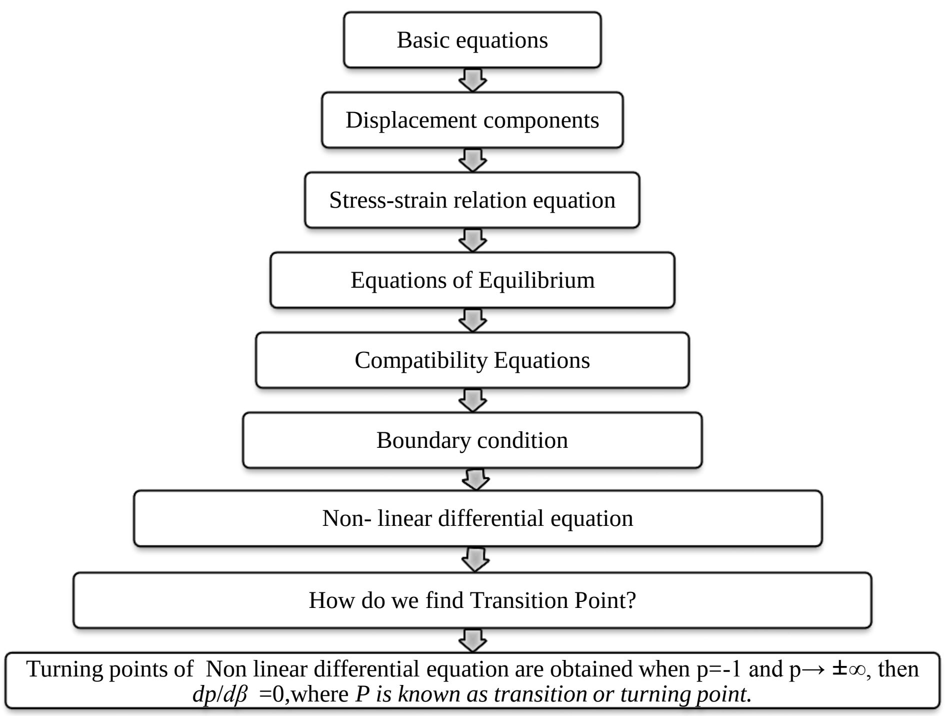
Seth (1962) proposed transition Theory and defined generalized strain measure. Seth's defined transition function for elastic-plastic and creep state. When a deformable solid is subjected to an external loading system, the solid first deforms elastically. If the loading is continued, plastic flow may set in, and if continued further, it gives rise to time dependent continuous deformation known as creep deformation. It may be possible that a number of transition states may occur at the same critical point, then the transition function will have different values, and the point will be a multiple point, each branch of which will then correspond to a different state. In general, the material from elastic state can go over into (i) plastic state, or to (ii) creep state, or (iii) first to plastic state and then to creep and vice-versa, depending upon the loading. All these final states are reached through a transition state. In the plane stress condition, the transition can take place either through the principal stresses τ_{rr} or $\tau_{\theta\theta}$ becoming critical or through the principal stress difference $\tau_{rr} - \tau_{\theta\theta}$ becoming critical. Hence we have to consider the following three cases: (*a*) Transition through τ_{rr}, (*b*) Transition through $\tau_{\theta\theta}$, (*c*) Transition through $\tau_{rr} - \tau_{\theta\theta}$. For each transition point, we have to determine the stresses and strains corresponding to the above three cases.

Appendix -III

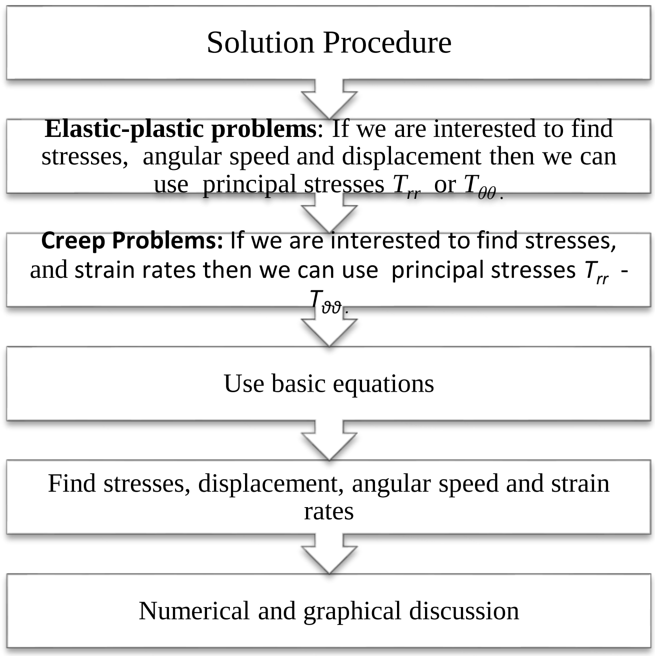
Details of Progress

At the initial stages of the project, intensive literature survey based on rotating disc subjected to various condition *e.g.* density, thickness, thermal gradient, non-homogeneity etc., for different materials. After that we construct displacement equation , Stress-Strain Relation, Equation of equilibrium, Boundary conditions and Critical points or Turning points was executed in a planned manner

1. **Construction of governing equation (*i.e.* Charts):** Displacement components, stress-strain relation, equation equilibrium equation, Non- linear differential equation, transition points and boundary conditions.



2. **Construction of solution Procedure of problems**



After that we discuss problems in disc as mentioned in <u>Annexure IV.</u>

Work-done/ Results Achieved

Paper 1. ***Elastic–Plastic infinitesimal deformation in a solid disk under heat effect by using Seth theory***

In this paper, we have studied the problem of elastic–plastic infinitesimal deformation in a solid disk due to heat source. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. It has been observed that radial stresses are maximum at the inner plastic zone and circumferential stresses are maximum **at the outer surface. Compressibility decreases values of stresses. Heat generation affects the** values of stresses and displacement to be increased. The results obtained have been illustrated by numerical results and has been compared with the heat generation case.

Paper 2. Elastic-plastic transitional stresses distribution and displacement for transversely isotropic circular disc with inclusion subject to mechanical load

In this paper, we have studied the problem of mechanical load in a thin rotating disc by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. Effect of mechanical load in a rotating disc with inclusion made of isotropic material as well as transversely isotropic materials increase the values of angular speed yield at the internal surface. With the introduction of mechanical load rotating disc made of Beryl material required maximum radial stress as compared to disc made of Mg and Brass materials at the internal surface.

Paper 3. Thermo Elastic-Plastic Deformation in a Solid Disk with Heat Generation Subjected to Pressure

Seth's transition theory is applied to the problem of elastic-plastic deformation in a solid disk due to heat source and subjected to pressure. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca's yield condition. The present solution is illustrated by numerical results and is compared with uniform heat generation case. This work provides the basis for a comprehensive investigation of the influence of non uniform heat generation subjected to pressure. Effect of heat increased values of stress for compressible material at the inner surface.

Paper 4. Infinitesimal deformation in a solid disk using Seth's transition Theory

In this paper, we have studied problems of solid rotating disk using transition theory. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. By specializing the results to perfectly plastic material, the usual statically determinate stress distribution is recovered but, since the plastic stress at the axis becomes infinite, these stresses are not meaningful. Compressible materials has made an increase in the values of plastic stresses at the centre of disk.

Paper 5. Determine the variation of Poisson ratios and thermal creep stresses and strain rates in an isotropic disc

In this paper, we have determined the variation of Poisson ratios and thermal creep stresses and strain rates in an isotropic disc using Seth transition theory. Thermal effect decreased value of radial stress at the internal surface of the rotating isotropic disc made of compressible material as well as incompressible material and this value of radial stress further much increases with the increase in angular speed. With the introduction of thermal effects, the maximum value of strain rates further increases at the internal surface for compressible materials as compared to incompressible material.

Paper 6. Steady Thermal Stresses in Solid Disk Under Heat Generation Subjected to Variable Density

Seth's transition theory is applied to the problem of stresses in a solid rotating disk under heat generation subjected to variable density by infinitesimal deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been seen that circumferential stress are maximum at the outer surface for incompressible material as compare to disk made compressible materials. Density variation parameter increases the value of circumferential as well as radial stress at the outer surface of solid disk for compressible and incompressible materials. The present solution is illustrated by numerical results and is compared with heat generation case.

Paper 7. Effect of mechanical load and thickness profile in a rotating disc by using Seth's transition theory.

In this paper efforts have been made to evaluate creep stresses and strain rate in a rotating disc with respect to changes in mechanical load and thickness profile. Seth's theory of transition has been used in this study. It has been observed that stresses increases with increase in mechanical load and maximum value of strain rate further increases at the internal surface for compressible materials. It is concluded that, rotating disc is likely to fracture by cleavage close to the shaft at the bore

Paper 8. Elastic-plastic distribution in a rotating disc with rigid inclusion and having thickness and density profile

The purpose of this paper is to analyze the elastic-plastic state of stress in a rotating disc with shaft having a thickness and density profile using Seth's transition theory. It has been observed that the rotating disc made of the compressible material with inclusion required higher angular speed to yield at the internal surface as compared to disc made of incompressible material and a much higher angular speed is required to yield with the increase in radii ratio. The thickness and density parameter decreases the value of angular speed at the internal surface of the rotating disc made of compressible as well as incompressible materials. The model proposed in this paper may be used in mechanical and electronic devices. The model also has extensive practical engineering application such as in steam and gas turbines, turbo generators, flywheel of internal combustion engines, turbojet engines, reciprocating engines, centrifugal compressors and brake discs.

Paper 9. Exact solution of rotating disc with shaft problem in the elastoplastic state of stress having variable density and thickness

The purpose of this paper is to study the elastioplastic state of stress in a rotating disc with a shaft having a thickness and density profile using Seth's transition theory. It has been observed that the rotating disc made of the compressible material with inclusion require higher angular speed to yield at the internal surface as compared to disc made of incompressible material and a much higher angular speed are required to yield with the increase in radii ratio. The thickness and density parameter decrease the value of angular speed at the internal surface of the rotating disc made of compressible as well as incompressible materials. The model proposed in this paper has applications in mechanical and electronic devices.

Appendix V

Summary of findings

The researchers had proposed to solve various problems of elastic plastic and creep analysis problems in a disc. Discs have a wide range of applications in engineering, such as high speed gears, turbine rotors, compressors, flywheel and computer's disc drive. The analytical procedures presently available are restricted to problems with simplest configurations. The use of rotating disc in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. The stresses and strains in discs made of various types of materials such as isotropic materials; transversely isotropic materials etc were calculated using Seth's Transition theory. This theory provides criteria to solve the problems by considering the stresses as a function of non linear partial differential equations. This theory has been successfully applied to solve various problems of elastic, plastic and creep deformation. In this project efforts have been made to solve various problems (see Appendix IV). The problems include obtaining stress and strain distribution in disc subjected to various parameters; such as variable density, variable thickness, pressure, mechanical load and attaching rigid shaft. It has been concluded that these parametric changes have a potent effect on the elastic plastic and creep deformation in a stationary and a rotating disc. Details of problems and solutions have been given in the research papers enclosed. The results obtained in this project have a potential impact on the design reliability of machines in heavy as well as light industries.

List of Publications

1. Elastic–Plastic infinitesimal deformation in a solid disk under heat effect by using Seth theory, International Journal of Applied and Computational Mathematics, **Springer Publisher**, DOI 10.1007/s40819-015-0116-9 , Nov., 13, 2015, pp. 1-13.
2. Elastic-plastic transitional stresses distribution and displacement for transversely isotropic circular disc with inclusion subject to mechanical load, Kragujevac Journal of Science, Serbia, Vol. 37, 2015, pp. 23-33. [Emerging Sources Citation Index **(ESCI)**]
3. Thermo Elastic-Plastic Deformation in a Solid Disk with Heat Generation Subjected to Pressure, Structural Integrity And Life, Vol. 15, No 3, 2015, pp. 135–142 [Emerging Sources Citation Index **(ESCI)**]
4. Infinitesimal deformation in a solid disk using Seth's transition Theory, International journal of electro mechanics and mechanical behaviour, Vol. 1: Issue 1, pp. 1-6.
5. Determine variation of Poisson ratios and thermal creep stresses and strain rates in an isotropic disc, Kragujevac Journal of Science, Serbia, Vol. 38, 2016, 15-28. [Emerging Sources Citation Index **(ESCI)**]
6. Steady Thermal Stresses in Solid Disk Under Heat Generation Subjected to Variable Density, Kragujevac Journal of Science, Serbia, Vol. 38, 2016, 5-14. [Emerging Sources Citation Index **(ESCI)**]
7. Effect of mechanical load and thickness profile in a rotating disc by using Seth's transition theory, AIP Conf. Proceeding, Vol. 1859, Issue 1, pp. 020024, USA, 2017.
8. Elastic - plastic distribution in a rotating disc with rigid inclusion and having thickness and density profile. Accepted for publication in Structural integrity and life, Vol. 18(2), 2018. [Emerging Sources Citation Index **(ESCI)**]
9. Exact solution of rotating disc with shaft problem in the elastoplastic state of stress having variable density and thickness. Accepted for publication in Structural integrity and life, Vol. 18(2), 2018. [Emerging Sources Citation Index **(ESCI)**]

Int. J. Appl. Comput. Math
DOI 10.1007/s40819-015-0116-9

ORIGINAL PAPER

Elastic–Plastic Infinitesimal Deformation in a Solid Disk Under Heat Effect by Using Seth Theory

Pankaj Thakur[1] · Satya Bir Singh[2] · Shruti Sawhney[1]

Abstract Seth's transition theory is applied to the problem of elastic–plastic infinitesimal deformation in a solid disk due to heat source. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. Future work may be directed to the transient heat generation case under the plane stress condition. It has been seen that radial stresses are maximum at the inner plastic zone and circumferential stresses are maximum at the outer surface. Compressibility decreases values of stresses. Heat generation affects the values of stresses and displacement to be increased. The present solution is illustrated by numerical results and is compared with the heat generation case.

Keywords Transition · Solid disk · Heat source · Deformation · Stresses · Yielding · Displacement

Introduction

Rotating disks have received a great deal of attention because of their wide use in many mechanical and electronic devices. They have extensive practical engineering applications such as in steam and gas turbines, turbo generators, flywheel of internal combustion engines, turbojet engines, reciprocating engines, centrifugal compressors and brake disks. Problems of rotating solid disks have been discussed performed under various interesting assumptions and the topic can be easily found in most of the standard elasticity and plasticity books [1–5]. The theoretical and experimental investigations on the rotating solid disks have been widespread attention due to the great practical importance in mechanical engineering. For a better utilization of the material, it is necessary to allow variation of the effective material or thick-

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ELASTIC-PLASTIC TRANSITIONAL STRESSES DISTRIBUTION AND DISPLACEMENT FOR TRANSVERSELY ISOTROPIC CIRCULAR DISC WITH INCLUSION SUBJECT TO MECHANICAL LOAD

Pankaj Thakur[1], **Satya Bir Singh**[2]

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ABSTRACT. Seth's transition theory is applied to the problems of mechanical load in a thin rotating disc by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. Effect of mechanical load in a rotating disc with inclusion made of isotropic material as well as transversely isotropic materials increase the values of angular speed yield at the internal surface. With the introduction of mechanical load rotating disc made of Beryl material required maximum radial stress as compare to disc made of Mg and Brass materials at the internal surface.

Keywords: Transversely isotropic, disc, shaft, stresses, displacement, yielding.

INTRODUCTION

This paper is concerned with finitesimal deformation of rotating thin circular disk made of transversely isotropic material. Finitesimal deformation consideration of problems of elasticity is often not easy to solve. There are many applications of rotating disks in science and engineering. As typical examples, we mention, steam and gas turbines, rotors, and flywheels. In the design of modern structures, increasing use is being made of materials which are transversely isotropic. The analysis of stress distribution in the circular disk rotating is important for a better understanding of the behavior and optimum design of structures. In the context of small deformation theory, the solutions for this problem of rotating disks made of isotropic material can be found in the most standard text books [1-4]. The analysis of thin rotating discs made of isotropic material has been discussed extensively by TIMOSHENKO and GOODIER [5] in the elastic range and by CHAKRABARTY [6] and HEYMAN [7] for the plastic range. Their solution for the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e. $T_{zz}=0$). A. P.

Thakur Pankaj[1], *Satya Bir Singh*[2], *Jasmina Lozanović Šajić*[3]

THERMO ELASTIC-PLASTIC DEFORMATION IN A SOLID DISK WITH HEAT GENERATION SUBJECTED TO PRESSURE

TERMO-ELASTOPLASTIČNA DEFORMACIJA ČVRSTOG DISKA SA IZVOROM TOPLOTE OPTEREĆENIM NA PRITISAK

Originalni naučni rad / Original scientific paper
UDK /UDC:
Rad primljen / Paper received: 25.06.2015

Adresa autora / Author's address:
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Keywords
- solid disk
- pressure
- heat
- stresses
- yielding
- displacement

Ključne reči
- čvrsti disk
- pritisak
- toplota
- naponi
- tečenje
- pomeranje

Abstract

Seth's transition theory is applied to the problem of elastic-plastic deformation in a solid disk due to heat source and subjected to pressure. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca's yield condition. The present solution is illustrated by numerical results and is compared with uniform heat generation case. This work provides the basis for a comprehensive investigation of the influence of non-uniform heat generation subjected to pressure. Effect of heat increased values of stress for compressible material at the inner surface.

Izvod

Setova teorija prelaznog stanja je primenjena na problem elastoplastičnog deformisanja čvrstog diska pod uticajem izvora toplote i pri opterećenju na pritisak. U ovom slučaju nije pretpostavljen kriterijum tečenja, a niti odgovarajući zakon protoka. Dobijeni rezultati su primenljivi na stišljiv materijal. Ako bi se zadao dodatni uslov nestišljivosti, onda izrazi za napone odgovaraju istim onim koji se izvode primenom uslova tečenja Treska. Dato rešenje je ilustrovano numeričkim rezultatima i dato je poređenje sa slučajem kada postoji uniformni izvor toplote. Rad pruža osnove za sveobuhvatno istraživanje uticaja promenljivog izvora toplote uz dejstvo pritisnog naprezanja. Uticaj toplote dovodi do povećanja napona u stišljivom materijalu na njegovoj unutrašnjoj površini.

INTRODUCTION

The use of rotating disks in machinery and structural applications has generated considerable interest in many problems in the domain of solid mechanics. There are many applications of such type of rotating disks, such as high speed gears, turbine rotors, flywheels, disk drives, etc. The stress distribution in an elastic-plastic rotating solid disk was first introduced by F. Laszlo, /1/. The usual approach for the determination of the elastic-plastic stress distribution is to apply the principle of momentum, Hooke's law, Tresca's yield condition and the condition of vanishing of radial stress at the outer surface of the disk. The first modern treatment for the elastic-plastic annular and solid disk with linear strain hardening has been given by Gamer, /2-4/. It was shown by Gamer /2/ that the analysis based on Tresca's yield condition for the elastic-perfectly plastic rotating solid disk is not meaningful. Accordingly, it was shown by Gamer /3/ that a meaningful solution for linear strain hardening can be obtained. In the analysis of Gamer, the plastic region of the disk in the elastic-plastic state consists of two plastic regimes with different forms of the yield condition, the inner being a corner regime and the outer a side regime of Tresca's hexagon. Thakur et al. /5/ investigated infinitesimal deformation in a solid disk by using Seth's transition theory. You et al. /6/ analysed elastic-plastic stresses in a rotating solid disk. Ahmet N., Eraslan et al. /8/ analysed rotating elastic-plastic solid disks of variable thickness having concave profiles. Seth's transition theory, /8/, does not acquire any assumptions like a yield condition, incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field, and has been successfully applied to a large number of problems /5, 8-12/. Seth /8/ has defined the generalized principal strain measure as:

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Infinitesimal Deformation in a Solid Disk Using Seth's Transition Theory

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Abstract
Seth's transition theory is applied to the problems of solid rotating disk. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. By specialising the results to perfectly plastic material, the usual statically determinate stress distribution is recovered but, since the plastic stress at the axis becomes infinite, these stresses are not meaningful. Compressible materials increased the values of plastic stresses at the centre of disk.

Keywords: *Transition, disk, deformation, stresses, yielding, displacement*

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INTRODUCTION

Rotating disks have received a great deal of attention because of their widely used in many mechanical and electronic devices. They have extensive practical engineering application such as in steam and gas turbines, turbo generators, flywheel of internal combustion engines, turbojet engines, reciprocating engines, centrifugal compressors and brake disks. The problems of rotating solid disks have been performed under various interesting assumptions and the topic can be easily found in most of the standard elasticity and plasticity books[1–5]. The theoretical and experimental investigations on the rotating solid disks have been widespread attention due to the great practical importance in mechanical engineering. For a better utilization of the material, it is necessary to allow variation of the effective material or thickness properties in one direction of the solid disk. Most of the research works are concentrated on the analytical solutions of rotating isotropic disks with simple cross-section geometries of uniform thickness and specifically variable thickness. The solution of a rotating solid disk with constant thickness is obtained by Gamer[6–7] taking into account the linear strain hardening material behavior. Gamer[6] found the "elastic-plastic deformation of the rotating solid disk under the assumptions of Tresca's yield condition, its associated flow rule and linear strain hardening. To obtain the stress distribution, they matched the plastic stresses at the same radius r = z of the disc. Seth's transition theory[8] does not acquire any assumptions like a yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field and has been successfully applied to a large number of the problems[9–11]. Seth[8] has defined the generalized principal strain measure as:

Kragujevac J. Sci. 38 (2016) 5-14. UDC 539.319

STEADY THERMAL STRESSES IN SOLID DISK UNDER HEAT GENERATION SUBJECTED TO VARIABLE DENSITY

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(Received February 3, 2016)

ABSTRACT. Seth's transition theory is applied to the problem of stresses in a solid rotating disk under heat generation subjected to variable density by infinitesimal deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been seen that circumferential stress are maximum at the outer surface for incompressible material as compare to disk made compressible materials. Density variation parameter increases the value of circumferential as well as radial stress at the outer surface of solid disk for compressible and incompressible materials. The present solution is illustrated by numerical results and is compared with heat generation case.

Keywords: disk, heat source, stresses, displacement, deformation, speed.

INTRODUCTION

Rotating disks have a wide range of application in engineering, such as steam and gas turbine, turbo generator, flywheel of internal combustion engines, turbojet engines, reciprocating engines, centrifugal Compressors, brake disk and shrink fit. The analytical elasticity-plasticity of such rotating disks of isotropic materials can be found in many books (TIMOSHENKO *et al.*, 1970, JOHNSON *et al.*, 1978), and the stress analysis in curvilinear orthotropic disks can also found in (LEKHNITSKII *et al.*, 1981) The theoretical and experimental investigations on the rotating solid disk have been widespread attention due to the great practical importance in mechanical engineering. For a better utilization of the material, it is necessary to allow variation of the effective material or thickness properties in one direction of the solid disk. Most of the research works are concentrated on the analytical solutions of rotating isotropic disks with simple cross-section geometries of uniform thickness and specifically variable thickness. The solution of a rotating solid disk with constant thickness is obtained by (GAMER, 1984, 1985) taking into account the linear strain hardening material behavior. (GAMER, 1984)

Kragujevac J. Sci. 38 (2016) 15-28. UDC 539.319

DETERMINE VARIATION OF POISSON RATIOS AND THERMAL CREEP STRESSES AND STRAIN RATES IN AN ISOTROPIC DISC

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ABSTRACT. Seth's transition theory is applied to the problem of thermal creep transition stresses and strain rates in a thin rotating disc with shaft having variable density by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. Thermal effect decreased value of radial stress at the internal surface of the rotating isotropic disc made of compressible material as well as incompressible material and this value of radial stress further much increases with the increase in angular speed. With the introduction of thermal effects, the maximum value of strain rates further increases at the internal surface for compressible materials as compare to incompressible material.

Keywords: strain rates, displacement, angular speed, disc, thermal stresses.

INTRODUCTION

Rotating discs provide an area of research and study due to their vast utilization in rotating machinery such as compressor, turbo generators, pumps, compressors, flywheels, shrink fits, automotive braking systems, ship propellers, computer disc drives, steam and gas turbine rotors. Theoretical investigation of the stresses and strain rates in annular discs rotating at high speeds have received widespread attention due to a large number of applications in mechanical and structural engineering. They are usually operated at relatively higher angular speed and high temperature. Therefore the prediction of long term steady state creep deformation is very important for these applications.

The classical theories of creep start with the assumptions of constitutive equations for creep and the classical theories of plasticity need an extra relation called the yield condition in addition to the flow rules. The description of the deformations in a solid subjected to external forces is thus given by a different set of equations for elastic, plastic and creep deformations. Solutions for thin isotropic discs can be found in most of the standard creep text books (LUBHAN, 1961; ODUIST, 1974; KARAUS, 1980; BOYLE, 1983; NABARRO, 1995; PENNY, 1995; HOFFMAN, 2012). In most of engineering application, the disc has to operate under elevated temperature and is simultaneously subjected to high stresses caused by disc rotation at high

Effect of Mechanical Load and Thickness Profile on Creep in a Rotating Disc by using Seth's Transition Theory

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Abstract. Rotating disc is a common component in turbines, rotors, compressors and other engineering components. In this paper efforts have been made to evaluate creep stresses and strain rate in a rotating disc with respect to changes in mechanical load and thickness profile. Seth's theory of transition has been used in this study. It has been observed that stresses increases with increase in mechanical load and maximum value of strain rate further increases at the internal surface for compressible materials. It is concluded that, rotating disc is likely to fracture by cleavage close to the shaft at the bore.

Keywords: Strain, Stresses, Disc, Load, thickness, material.

INTRODUCTION

Rotating disc plays an important role in machine design. Stress analysis of rotating discs has an important role in engineering design. Rotating discs are the most critical part of rotors, turbine motors, compressor, flywheels, gears, shrink fits etc. Various machines such as centrifuges and separators for division or filtration of suspensions and emulsions, gas and steam turbine engines, turbochargers, etc. incorporate in their construction the rotating parts of various shapes, which operate under inertial (centrifugal) force and surface loads. Problem of stress calculation in a rotating thin disk was considered for the first time in [1] report "On the equilibrium of elastic solids" The classical theories of creep start with the assumptions of constitutive equations for creep and the classical theories of plasticity need an extra relation called the yield condition in addition to the flow rules. The description of the deformations in a solid subjected to external forces is thus given by a different set of equations for elastic, plastic and creep deformations. The use of rotating disc in machinery and structural applications has generated considerable interest in many problems in the domain of solid mechanics. Solutions for thin isotropic discs can be found in most of the standard creep textbooks [2 -4]. Wahl [7] has investigated creep deformation in rotating discs assuming small deformation, incompressibility condition, Tresca yield criterion, its associated flow rule and a power strain law. Mendelson [6] used an iterative scheme to obtain the thermo-plastic solution for rotating disks based on the Lame's solution. Gamer [7-8] presented more modern treatment for an elastic plastic annular disk with linear hardening behavior subjected to external pressure. You et al. [9, 10] have studied rotating disks of variable thickness and density, using a unified numerical method based on a polynomial plastic stress- strain relation and Von-Mises's yield criterion. Hojjati and Jafari [11-13] studied the elastic and elastic-plastic analyses of non-uniform thickness and density rotating disk under only centrifugal body loadings by using three semi-exact methods namely the variational iteration method, homotopy perturbation method, and adomian's decomposition method. Gupta et. al. [14] analyzed creep transition in a thin rotating disc with rigid Inclusion by using Seth transition theory. Thakur *et al*. [17] investigated thermal creep stresses and strain rates in a circular disc with a shaft having variable density by using Seth theory. Seth's transition theory does not acquire any assumptions like a yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at critical

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Dear author(s),

Thank you for submitting the paper titled:

EXACT SOLUTION OF ROTATING DISC WITH SHAFT PROBLEM IN THE ELASTOPLASTIC STATE OF STRESS HAVING VARIABLE DENSITY AND THICKNESS, by P. Thakur, S.D. Shahi, S.B. Singh, F.S. Emmanuel.

After a positive double-blind peer review, the paper has been accepted for publication in the following issue of the scientific journal *STRUCTURAL INTEGRITY AND LIFE*, Vol.18, No.2. Technical editing and print-out preparation are yet to be completed.

The issue of STRUCTURAL INTEGRITY AND LIFE – 2/2018 shall most probably be published by the end of August 2018.

Please do not hesitate to contact me or any member of the editorial board should you have any concerns regarding your article and our journal.

Sincerely,

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